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BURIED INSULATED ANTENNAS.(U)

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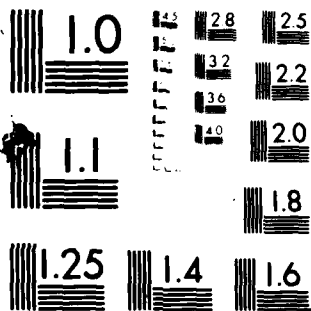
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ROYAL SIGNALS & RADAR  
ESTABLISHMENT

BURIED INSULATED ANTENNAS

Author: D J Brammer

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ROYAL SIGNALS AND RADAR ESTABLISHMENT

Memorandum

Title: BURIED INSULATED ANTENNAS

Author: D J Brammer

Date: October 1981

SUMMARY

Two methods of treating the buried insulated antenna are described, transmission line and moment methods. These two methods are compared. Calculations are performed and results given at HF for a range of antenna lengths and insulation thicknesses, giving the gain, front to back ratio and impedance.



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BURIED INSULATED ANTENNAS

D J Brammer

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1 INTRODUCTION

The type of antenna considered is shown in figure 1 and consists of a buried central conductor probably covered in a layer of dielectric insulation for some part of its length. This type of antenna is described in reference [1]. The current can be thought of as radiating an electric field which passes to the surface and is refracted towards the interface on passing through the interface. Normally most of interest is the field on the earth's surface and therefore also the lateral wave which can be thought of as travelling along the interface. The angle at which the rays travel to the surface will in this case be the critical angle. Reference [2] suggest that tilting the antenna at right angles to these critical rays would increase the signal. Due to the difficulty of burial of long wires, only shallow burial and antennas parallel to the interface are considered.

The central conductor, the insulation and the higher permittivity lossy ground form a transmission line which will support a travelling wave. Stratton [3] page 547 gives an equation for the permissible values of the wavenumber of the line  $K_L$ . If the radius of the insulation is much less than the wavelength, and the complex dielectric constant changes appreciably across the insulator ground boundary, only the lowest order mode is non-evanescent. Lee et al [1] give approximate formulae found in section 2 for the wavenumber  $K_L$ .

So that there shall be progressive reinforcement of the wave at the surface, the nearer  $K_L$  is to the wavenumber  $K$  of the lateral wave (same as the wavenumber in free space) the better. The effect of insulation thickness is dealt with in section 2.

Another factor of importance is the front to back ratio. Although in figure 1 the waves are shown travelling to the right, depending on the character of the current on the central conductor there can be a similar signal to the left. If the line wavenumber is fairly close to freespace then a wave travelling from left to right down the line sets up the phase of the current on the line to favour a lateral wave travelling from left to right ie the phase of rays reaching the surface will be that of a wave travelling from left to right. If at the end of the line the travelling wave is reflected backwards, there will be an appreciable signal in the backward direction. Also if the line is appreciably less than a wavelength long the backward signal will be high.

Another method available for the treatment of the problem will be discussed in section three, the moment method. In this technique the buried antenna is divided into elements and the reaction of one element on another in the presence of the interface above and the dielectric insulator is calculated. This method allows a more realistic antenna configuration to include earth spikes, loading and tapering of the insulation of the antenna as well as including the effect of the interface.

These two methods allow design parameters such as insulation thickness, depth of burial to be determined.

## 2 TRANSMISSION LINE TREATMENT

As stated in the introduction the central conductor, the insulation and the outer conducting dielectric (the earth) form a transmission line. The equation on page 547 of Strattons book [3] will give the value of the wavenumber. Lee et al [3] from a different approach give two approximate formulae for the wavenumber  $K_L$ :-

$$K_L = K_2(1 + H_0(K_4 b) / (K_4 b \ln(b/a) H_1(K_4 b)))^{1/2} \dots\dots\dots 1$$

Accurate for  $\left| K_4^2 / K_2^2 \right|$  greater than 16  
 $K_2 b$  much less than 1

$$K_L = K_2((K_4^2(H_0(K_4 b) + K_4 b \ln(b/a) H_1(K_4 b))) / (K_2^2 H_0(K_4 b) + K_4^2 K_4 b \ln(b/a) H_1(K_4 b)))^{1/2} \dots\dots\dots 2$$

Accurate for  $\left| K_4^2 / K_2^2 \right|$  as low as 1.5  
 $K_2 b$  much less than 1

where  $K_2$  is the insulation wavenumber

$K_4$  is the ground wavenumber

$a$  is the central conductor radius

$b$  is the insulation radius

$H_0, H_1$  are Hankel functions of the first kind of zero and first order respectively.

Both these formulae may be deduced from the formula in Stratton's book by suitable approximations. For example by replacing the Bessel functions  $J_0, Y_0, J_1, Y_1$  by the first terms in their expansions for small arguments, formula 1 results. By computing exact values and comparing them with values from formula 2, the approximate values using this formula have been found to be very accurate for typical earth values and frequencies in the HF range. Some typical values for the line wavelength and attenuation using this second formula for  $K_L$  are shown in table 1. Also shown for air insulation at 4 and 16 MHz for ground parameters of permittivity 16 and conductivity .01 S/M.

TABLE 1

	4 MHz			16 MHz		
Cladding thickness	$\lambda_{line}$ metres	$\lambda_{beat}$ metres	loss dB/M	$\lambda_{line}$ metres	$\lambda_{beat}$ metres	loss dB/M
none	13.28	16.14	2.9	4.44	5.83	3.81
.5 mm	17.41	22.68	.79	5.76	8.32	1.93
1 mm	21.41	29.98	.46	6.66	10.3	1.45
2 mm	26.57	41.17	.28	7.85	13.5	1.05
5 mm	34.28	63.17	.16	9.70	20.11	.719
10 mm	40.17	86.58	.12	11.15	27.56	.574
25 mm	47.54	130.0	.09	13.00	42.51	.455
infinity	74.98	inf.	.00	18.74	inf.	.000

Since the size of the insulation is a small fraction of a wavelength it would be expected that higher order modes would be rapidly attenuated.

Consider an antenna such as shown in figure 1 driven at one end of the insulation against an earth spike. The first part of the line will set up a lateral wave travelling along the surface with a wavenumber  $K$  (of free space) but since the wave is travelling along the line with wavenumber  $K_L$  the relative phase of the line and the lateral wave goes up and down with a wavelength beat given in the table where

$$1/\lambda_{beat} = 1/\lambda_{line} - 1/\lambda_{air}$$

This effect is clearly shown in figures 2 and 3. These figures show the relative received power at the surface for the stated length of buried insulated antenna. It is assumed a unit wave has been launched down the antenna and the receiving point is on the surface in a vertical plane containing the antenna in a direction away from the antenna the same as the direction of the travelling wave.



Values plotted in figures 2 and 3 are calculated from

$$20 \log_{10} \left( \left| \int_0^L e^{i(K_L - K_{air})d} dl \right| \right) / \left( \left| \int_0^1 e^{i(K_L - K_{air})d} dl \right| \right)$$

Where  $K_L$  is the wavenumber of the line,  $K_{air}$  the wavenumber in air.

This is the signal relative to a 1 metre length.

The thicker the cladding the larger the eventual signal will be, since the longer the length of line before phase reversal occurs. After the maximum is reached the signal falls away again. The effect of the attenuation in the line is to damp the oscillation shown and the signal eventually falls to a constant value unaffected by any change in length. In the antenna is short, say for example 10 metres, and since all thicknesses of cladding have the same efficiency curve at first, it would be pointless to increase the insulation beyond a certain thickness indicated by these curves. The effect of a plastic insulator instead of an air one is seen to decrease the beat wave length and so decrease the size of the eventual signal maximum.

The front to back ratio can also be calculated in a similar simple way and is shown in figures 4 and 5. Short lengths do not show any back to front ratio.

The backward signal behaves in a manner similar to the forward signal but since the phase of the wave on the line and lateral wave are in opposition the beat wavelength is much more rapid and is given by:-

$$1/\lambda_{beat} = 1/\lambda_{line} + 1/\lambda_{air}$$

Again due to attenuation there is not complete cancellation and the backward signal falls to a small value which is not zero. The front to back ratio therefore rises cusp-like to a maximum at the first minimum of the backward signal given by the above formula.

The effect of depth of burial can be assessed by noting that for a reasonable earth dielectric constant the angle of travel to the surface is nearly vertical and these rays suffer an attenuation given in table 1 of 2.9 dB/m at 4 MHz and 3.88 dB/M at 16 MHz for the ground parameters given.

### 3 MOMENT METHOD TREATMENT

In the previous transmission line treatment there are a number of effects that are ignored ie the effect of the earth's surface on the transmission line and the practical excitation of the line itself which may be uncertain, as there is no definite outer conductor to connect to. To assess the accuracy of the transmission line treatment a moment method calculation was carried out. As is usual with the moment method [4] the central conductor was divided into a number of elements along its length. Each one is assigned an unknown current. The electric field at one element due to the current in another is calculated using the routines in reference [5]. In addition the effect of the dielectric cladding was included by a method due to Richmond [6], described more fully in

an appendix. The boundary condition of zero tangential electric field at the conductor surface was imposed at the centre of each element. Solving the resultant set of simultaneous equations enables the current on the conducting wire to be calculated. Knowing this current the field on the surface of the earth at a distance can be calculated using the methods described in reference [7]. This essentially consists of adding the attenuation of a wave travelling to the surface to the attenuation of the lateral wave travelling along the surface.

The current in the central conductor is shown in figure 6, the bottom part of the figure shows schematically the arrangement of conductors and insulation. There is a central insulated part of 20 metres in length and two bare sections of 10 metres each end which serve as earth spikes. The antenna is driven by a unit voltage generator at the junction of the left hand bare section with the insulated part ie at the 10 metre point on the length scale. The two curves above this show the relative amplitude and phase respectively of the current in the conductor. To the left of the driving point the current decays exponentially fairly rapidly with a phase that changes linearly with distance. This clearly demonstrates an attenuated travelling wave down the earth spike. The complex wavenumber for this wave, if the wire is thin, will be the complex wavenumber for waves travelling in the conducting dielectric earth. To the right of the driving point another wave travels with less attenuation and a less rapid phase change with distance down the insulated part of the wire, thus exhibiting transmission line behaviour. Note the ripples in amplitude at the end of the insulated part indicating a reflection at the end and a reflected wave travelling back down the line. In this case the amplitude reflection factor deduced from the ripple amplitude is about 15%.

From the plots of phase and relative amplitude of the current the line wavelength and attenuation given by the moment method can be compared with the transmission line calculation.

TABLE 2

Moment Method			Transmission line	
FREQ	Wavelength	Loss	Wavelength	Loss
MHz	Metres	dB/M	Metres	dB/M
4	32.	-	34.28	.164
11	13.7	.61	13.6	.60
16	9.8	.81	9.7	.719

The two methods agree quite well.

Also printed at the bottom of figure 6 is the field in the forward direction (along the antenna from left to right) on the surface of the earth at a range of 1000 metres. The front to back ratio and impedance at the driving point are also given.

A 15% reflection at the insulation end could degrade an infinite front to back ratio to 16.5 dB. The reflection at the end is therefore only really important for high front to back ratios, which as can be seen from

figures 4 and 5 occurs only for narrow ranges of antenna length. This reflection at the end of the insulation can be reduced by matching the insulated transmission line part to the bare part. A series of resistances were found to be ineffective as they tended to be shunted by the conducting earth. Another method shown in figure 7 is to taper the insulation down to zero. As can be seen from the plots of the amplitude and phase of the current the background reflected wave has been reduced. Due to the relatively long length of the antenna the front to back ratio is only 6.6 dB in figure 6 and reducing the backward reflection would have little effect. Increasing the effective length of the antenna, a much more critical factor has in fact reduced the front to back ratio.

The left hand bare section or earth spike of figure 7 shows a rapid exponential decay to a small value, the shorter earth spike shown in figure 8 is adequate. The bare section at the end of the tapered section is still needed as is evident from figure 9.

Since the moment method calculations exhibit transmission line behaviour it would be expected that such calculations agree with those of figures 2-5. To confirm this a set of different lengths were analysed at 4 MHz, one of the set is shown in figure 10. Here it is seen that the taper is only partially effective in reducing the back reflection, but as previously explained this is only important for high back to front ratios.

Table 3 gives the front to back ratio as a function of the total insulator length including the tapered section.

TABLE 3

Length metres	F/B ratio dB	Gain relative to 10 m in dB
10 m	1.4	0
14 m	3.3	3.64
20 m	13.1	4.24
24 m	16.4	3.52

These results compare quite well with figures 2 and 4 the transmission line calculations.

The 4 metre tapered section is about .3 transmission line wavelengths at 11 MHz and appears adequate, whereas at 4 MHz it has fallen to .12 wavelengths and is clearly too short. This suggests a quarter wavelength is about the minimum length.

Results at 2 MHz require a longer taper and a set of results was calculated which illustrates the effect of changes in taper length and also the effect of earth spikes. The transmission line wavelength is about 64 metres for the 2 MHz case, so that the taper should be at least 16 metres. Table 4 gives front to back ratios as a function of length and taper length:-

Figure 11 shows an antenna with 44 m of insulation at 2 MHz, there is an appreciable reflection but despite this the front to back ratio is still 13.1 dB. The reflection from the end is reduced by the 10 m taper shown in figure 12. Figure 13 is an amplitude and phase diagram for this antenna. Starting at the origin a small vector is added corresponding to each current element along the line, one diagram for the forward and one for the backward direction. The section from the origin to the first slanting arrow is the signal from the first bare part or earth spike. The section to the next arrow is the signal from the 5 mm thick insulated part, the section between the second and third slanting arrows is the taper and finally the signal from the second (right hand side) earth spike which is negligible. Although the backward signal diagram is far from the ideal circle the resultant backward signal is not large.

In figure 14 the taper is longer so reducing the backward signal in the transmission line but due to the signal from the earth spike at the beginning of the insulated part the resultant backward signal is small and the front to back ratio large.

Figure 15 shows a longer taper which has actually reduced the front to back ratio as the amplitude and phase diagram for the backward signal has wound up too tightly leaving the signal from the first earth spike remaining. Running this earth spike at right angles to the antenna could reduce this remaining signal. Reducing the backward signal therefore, may not always be a good thing.

Table 4 summarises the 2 MHz results. The variation of the front to back ratio for antennas with 14 metre tapers ie the 24, 34 and 44 metre lengths agree quite well with the transmission line calculations at the top left of figure 4.

TABLE 4

Total Insulation length metres	Taper length metres	F/B dB	Field at 1000 m for 1 mW drive power	
24	14	3.0	5.6	
34	14	8.2	6.9	
44	0	13.1	6.0	(Fig 11)
44	10	19.7	6.1	(Fig 12, 13)
44	14	25.1	6.1	(Fig 14)
44	20	15.8	6.0	(Fig 15)

#### 4 CONCLUSIONS

Both transmission line methods and moment methods can be used to analyse buried antennas. Factors relevant to buried antenna design are:-

##### 4.1 LENGTH

Reference to figures 1 to 4 indicates what front to back ratio and relative gain can be expected for various lengths and frequencies.

#### 4.2 INSULATION THICKNESS

Given a value for the length it can be seen from figures 1 to 4 there is a thickness above which it is not worth increasing the insulation thickness.

#### 4.3 DEPTH OF BURIAL

The loss is given in table 1 and is around a few dB per metre of burial. This would not be significant for shallow burial.

#### 4.4 LENGTH OF EARTH SPIKES

Again table 1 gives the attenuation for a bare wire which is quite high. The length should be chosen to obtain a reasonable attenuation say 80%. A length longer than this is not worthwhile.

#### 4.5 TAPERING OF THE INSULATION

The signal reflected off the end of the effective transmission line part of the antenna can be reduced by tapering the insulation thickness to zero.

#### 4.6 PLASTIC INSULATION

This is appreciably worse than the air insulated case from the gain and front to back ratio point of view, as can be seen from figures 2 to 5.

5 REFERENCES

- 1 K M Lee, T T Wu, and R W P King: "Theory of an Insulated Linear Antenna in a Dissipative medium", Radio science. 12, pp 195-203 (1977).
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- 4 R F Harrington: "Field Computation by Moment Methods", Macmillan, New York, 1968.
- 5 R J Lytle and D L Lager: "Fortran Subroutines for Sommerfeld Integrals Under Antennas", Lawrence Livermore Laboratory, California, USA.
- 6 J H Richmond: "Radiation and Scattering by Thin Wire Structures in the Complex frequency domain", NASA Report No CR-2396, May 1974.
- 7 R J Lytle, E K Miller and D L Lager: "A Physical Explanation of Electromagnetic Surface wave Formulas", Radio Science, Vol 11, No 4, pp 235-243, April 1976

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## APPENDIX

### THE RICHMOND DIELECTRIC CLADDING APPROXIMATION

This is described in reference 6. The wire, cladding and surrounding earth are shown in figure A1. The first step is to rearrange Maxwells equation for inside the cladding region of dielectric constant  $\epsilon_2$ .

$$\begin{aligned}\text{Curl } \vec{H} &= \vec{J} + \dot{\vec{D}} \\ &= \vec{J} + (\epsilon_2 - \epsilon_1 + \epsilon_1) \dot{\vec{E}} \\ &= \vec{J} + \epsilon_1 \dot{\vec{E}} + (\epsilon_2 - \epsilon_1) \dot{\vec{E}}\end{aligned}$$

ie the cladding can be replaced by an equivalent current

$$\vec{J}_{\text{equiv}} = j\omega(\epsilon_2 - \epsilon_1)\vec{E}$$

assuming a time dependance of  $e^{j\omega t}$

near a wire the field is perpendicular to the wire. The charge on a length

$$dl \text{ of wire is } -\frac{\partial I}{\partial l} \times \frac{dl}{j\omega}$$

the radial field in the direction  $\hat{S}$  is given by

$$E_{\text{RADIAL}} = \frac{-\frac{\partial I}{\partial l} \hat{S}}{\epsilon_2 j\omega \cdot 2\pi S}$$

$$\begin{aligned}\text{and so } J_{\text{equiv}} &= \frac{\epsilon_2 - \epsilon_1}{\epsilon_2} \cdot \frac{1}{2\pi} \cdot \left(-\frac{\partial I}{\partial l}\right) \cdot \frac{\hat{S}}{S} \\ &= AI' \frac{\hat{S}}{S} \quad \text{where } A = -\frac{(\epsilon_2 - \epsilon_1)}{2\pi\epsilon_2}\end{aligned}$$

$$I' = \frac{\partial I}{\partial l}$$

referring to Figure A2

the current in the sector of angle  $\phi$

$$= AI' d\phi$$

for a disc  $\phi = 2\pi$

$$\text{Total Current} = 2\pi AI' dl$$

knowing this current the field at any point due to it may be calculated, note it is independent of  $S$  the radius.

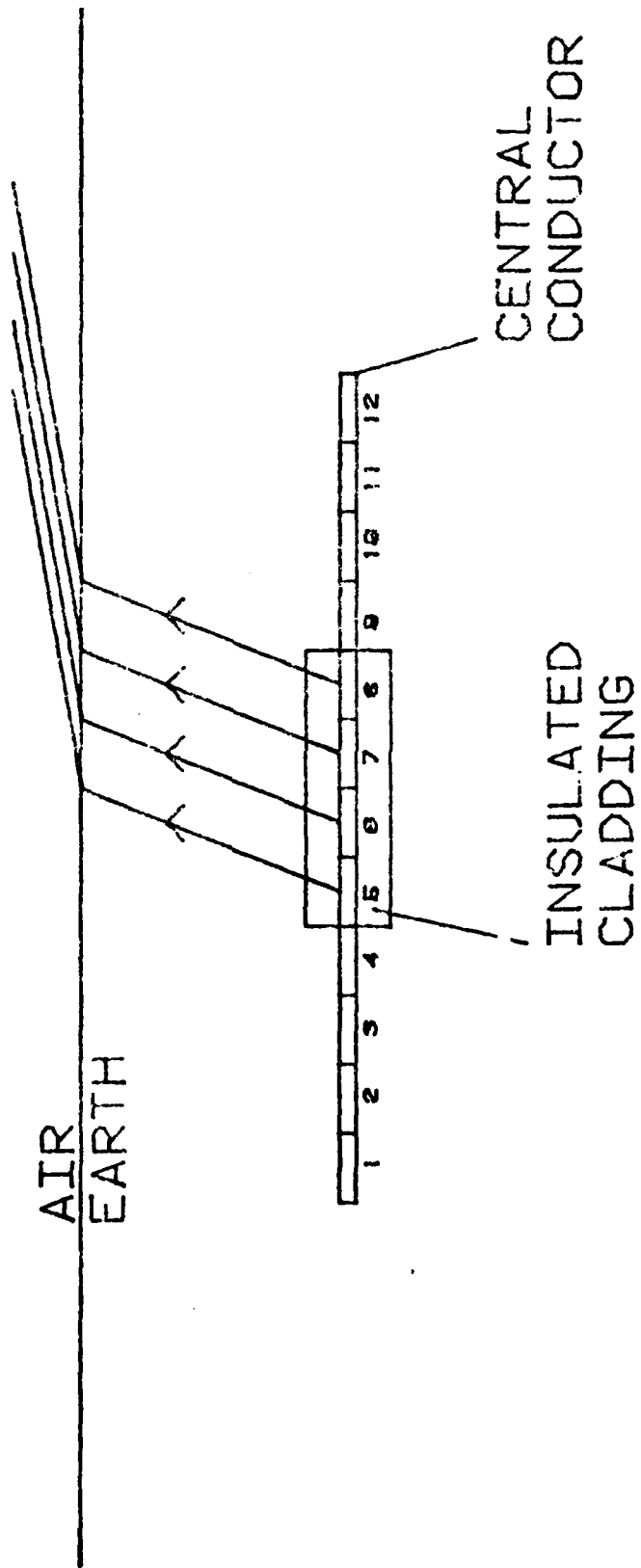


FIG. 1



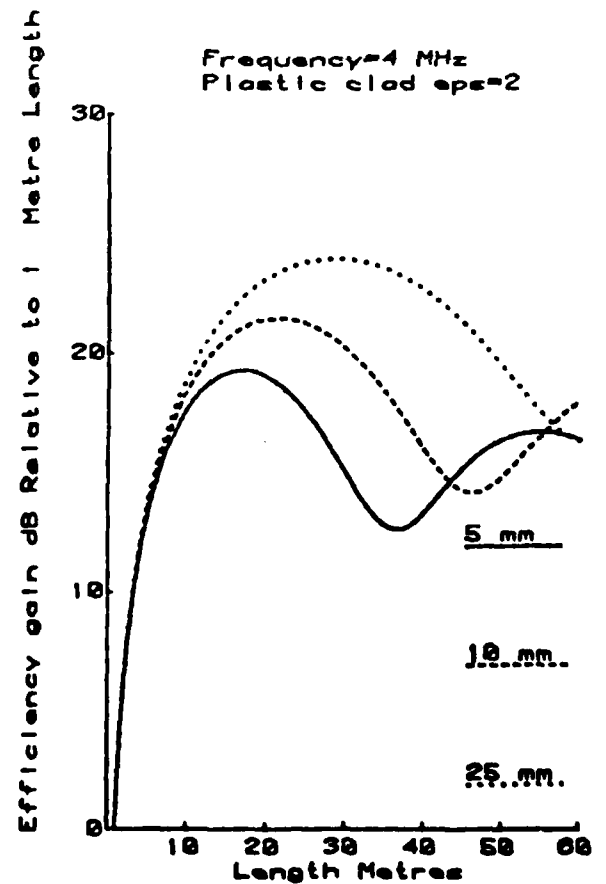
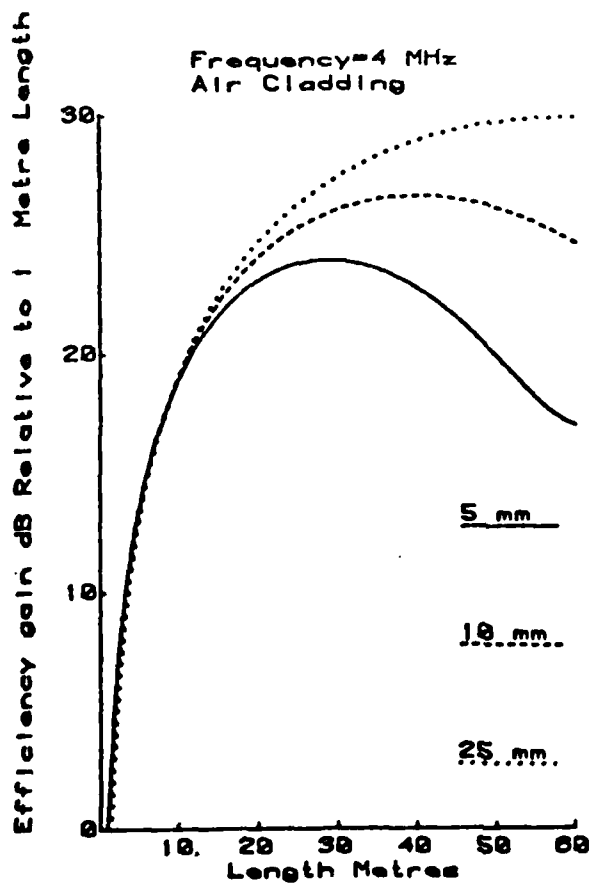
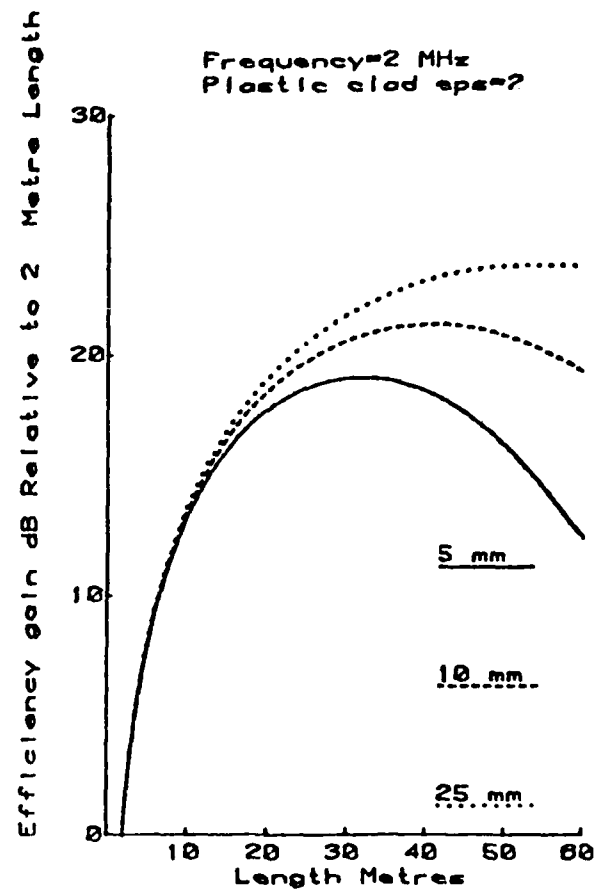
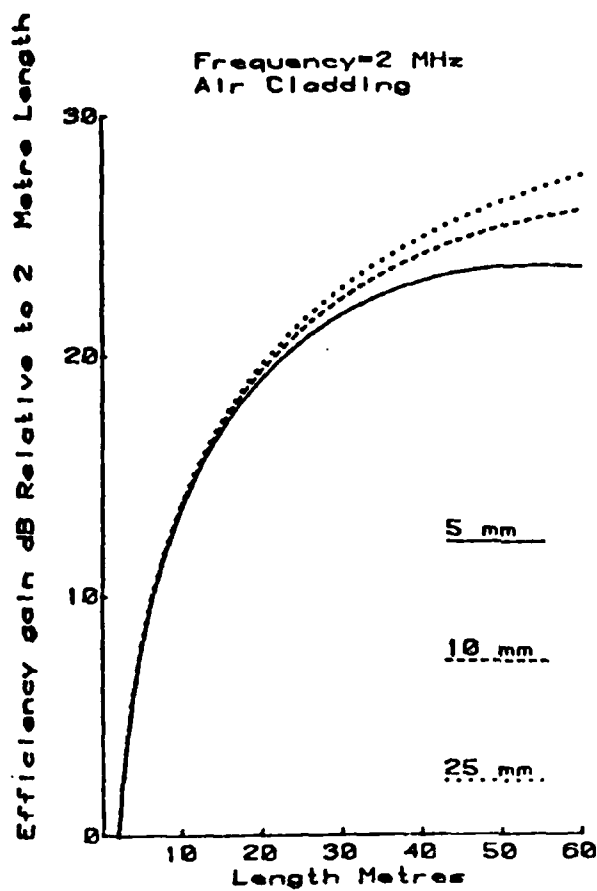


Figure 2 Variation of gain with length at 2 MHz and 4 MHz for air and plastic coatings of various thicknesses.

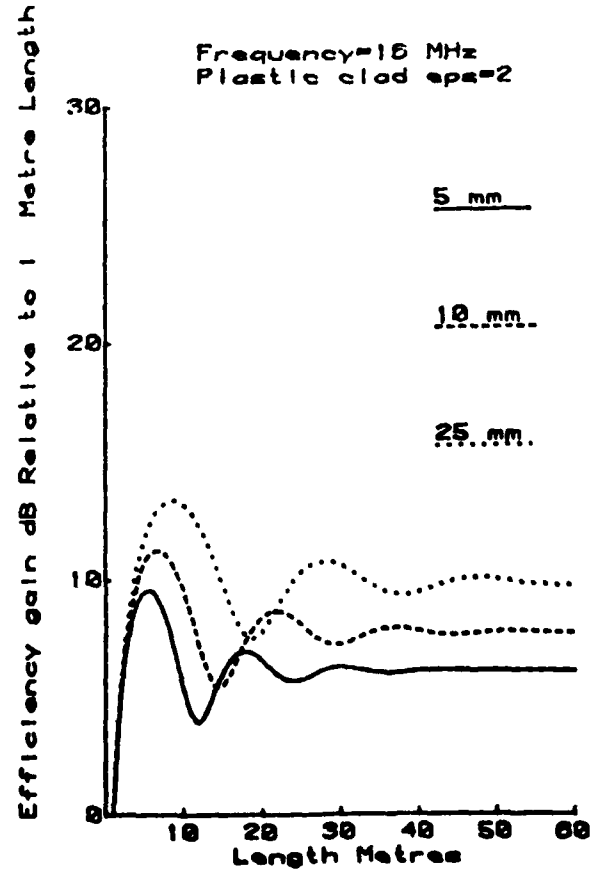
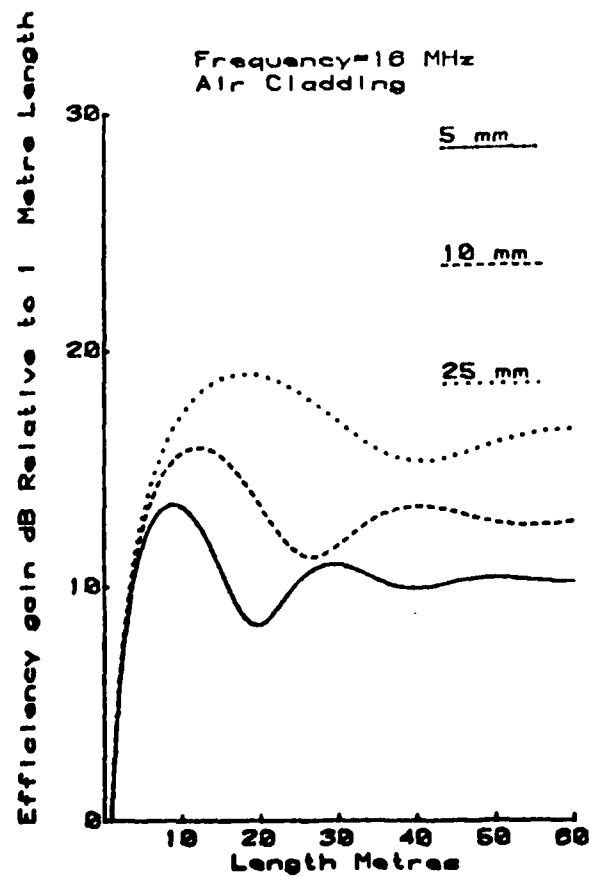
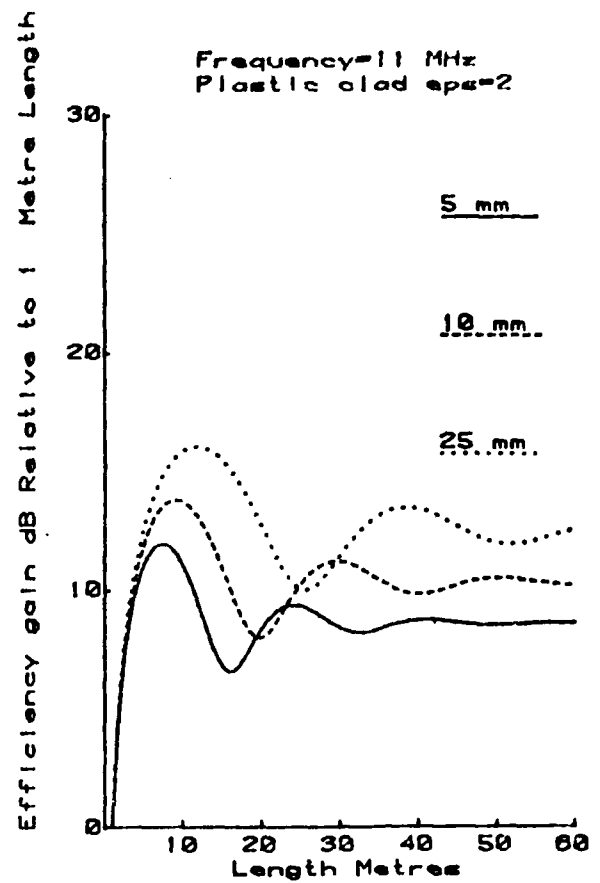
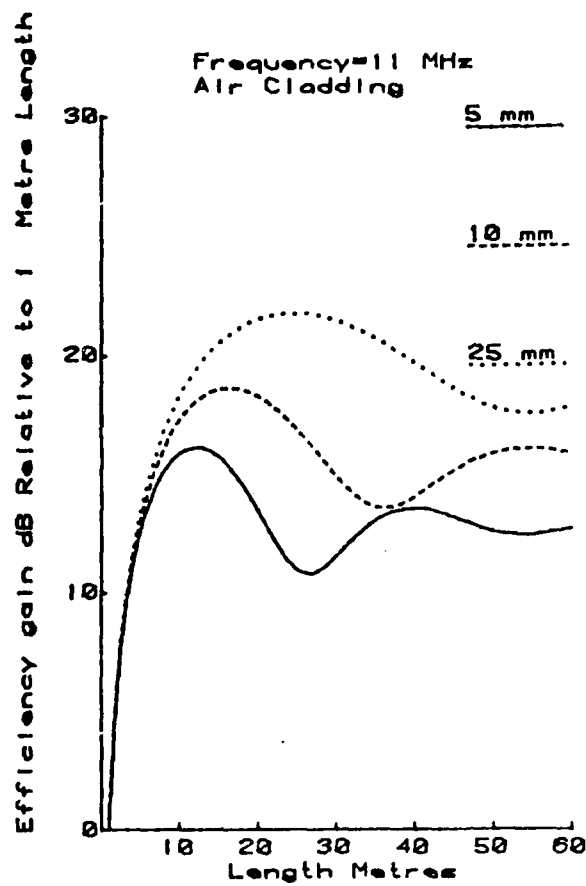


Figure 3 Variation of gain with length at 11 MHz & 16 MHz for air and plastic coatings of various thicknesses.

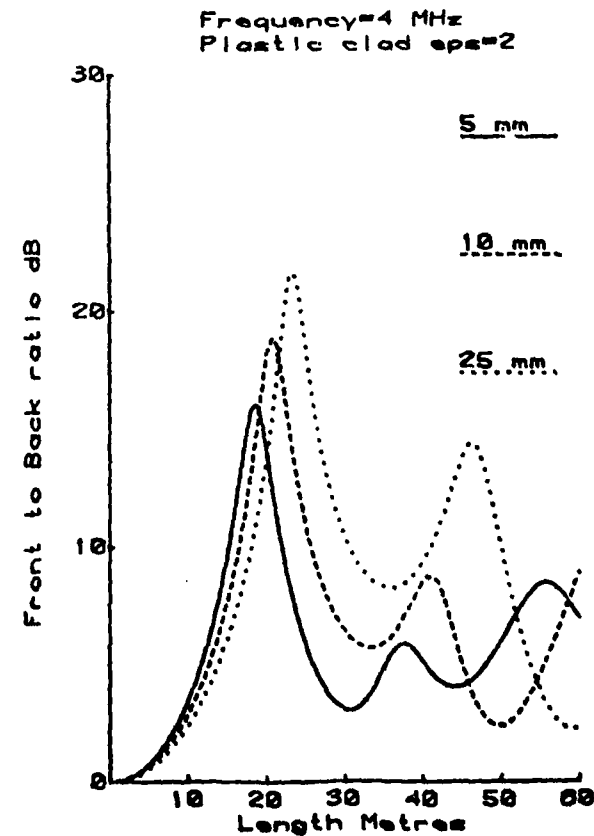
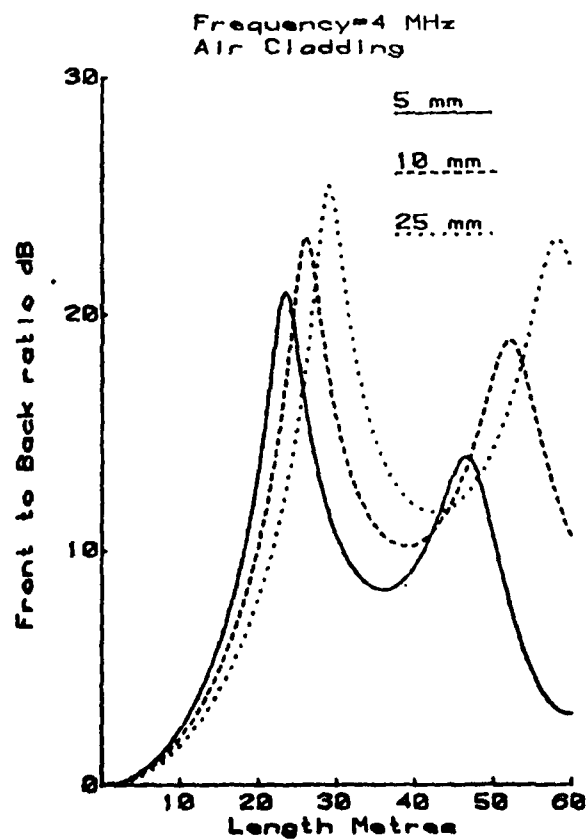
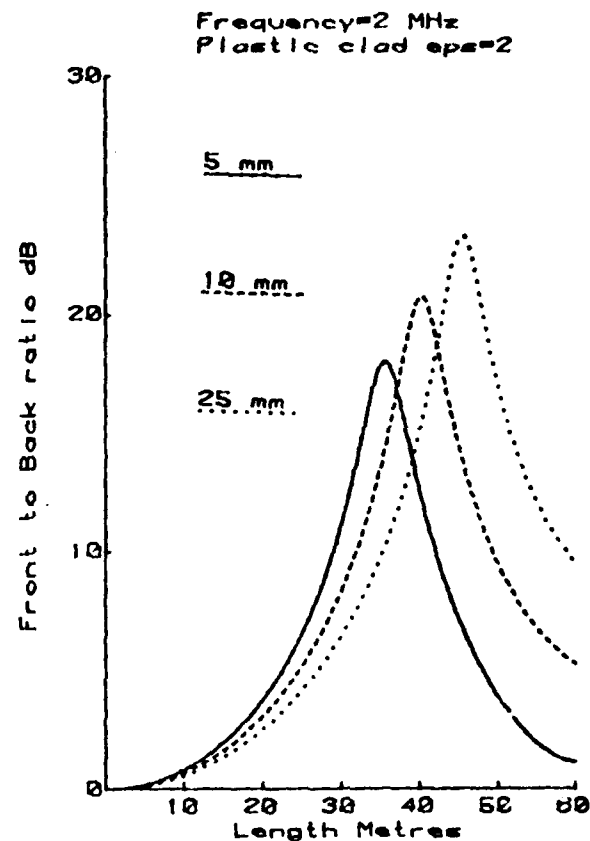
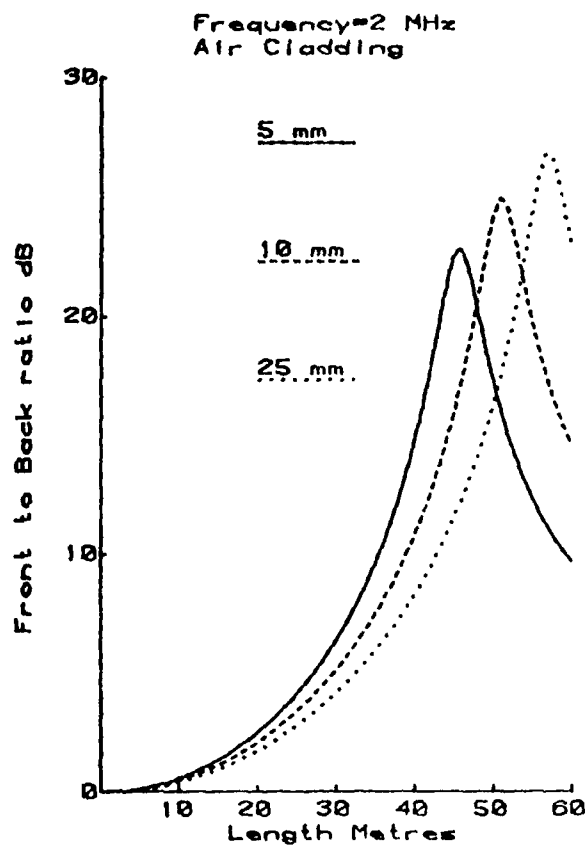


Figure 4 Variation of front to back ratio with length at 2 MHz & 4 MHz for air and plastic coatings of various thicknesses.

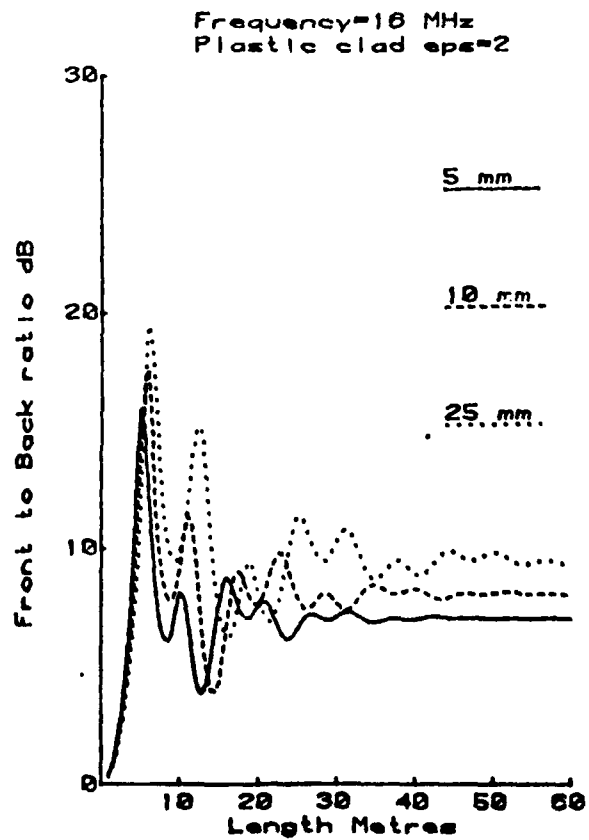
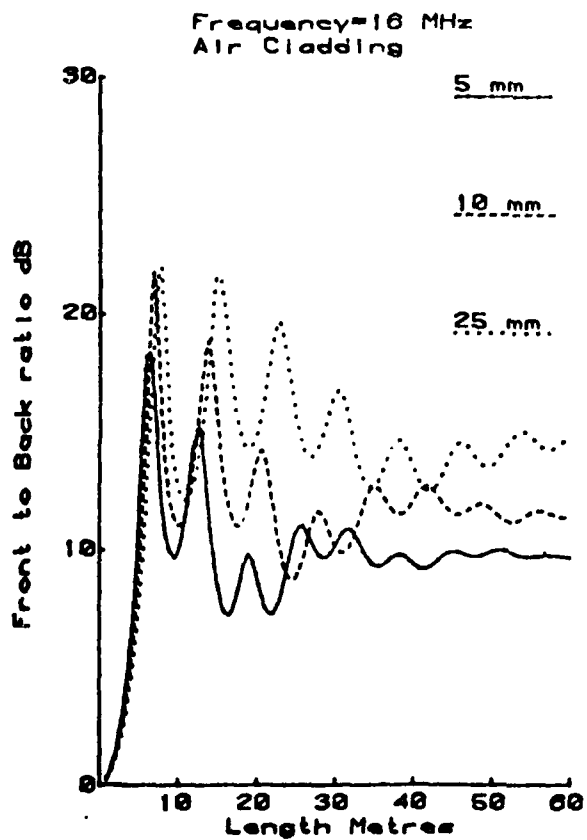
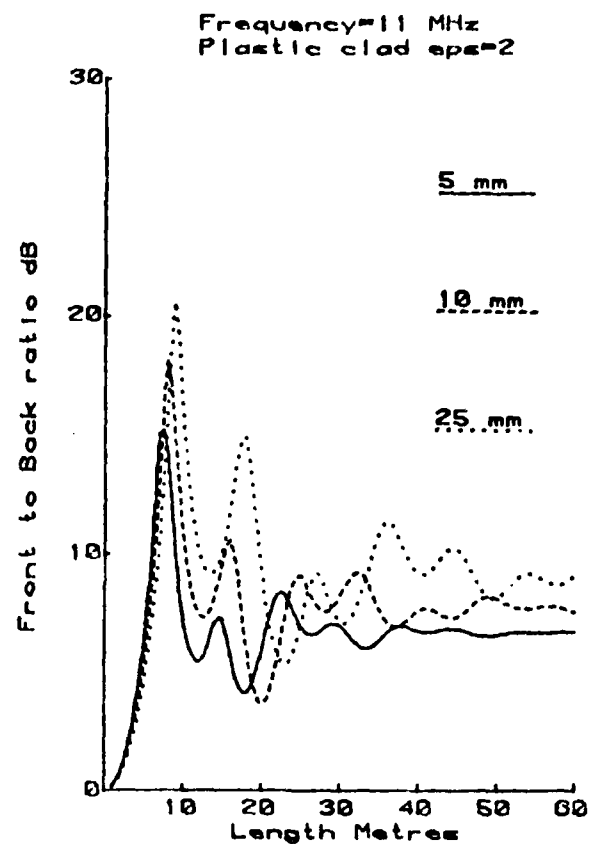
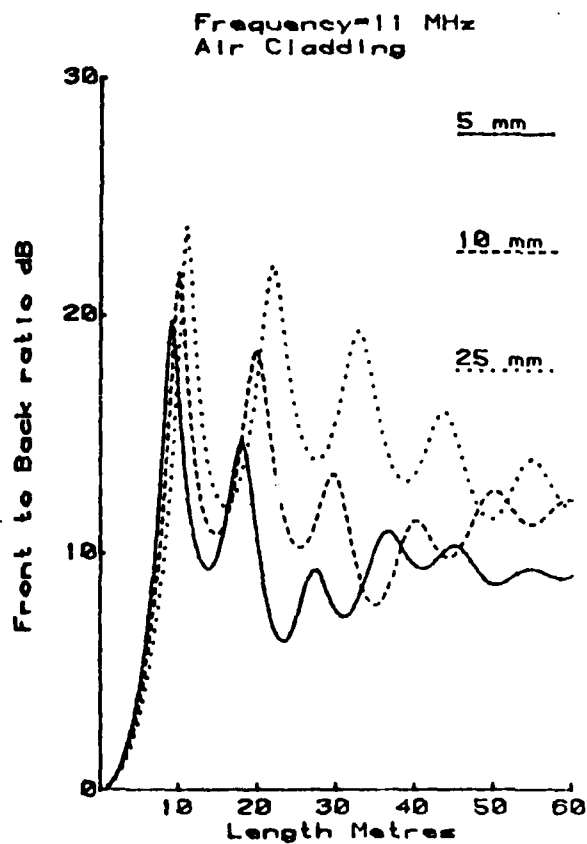


Figure 5 Variation of front to back ratio with length at 11 MHz & 16 MHz for air and plastic coatings of various thicknesses.

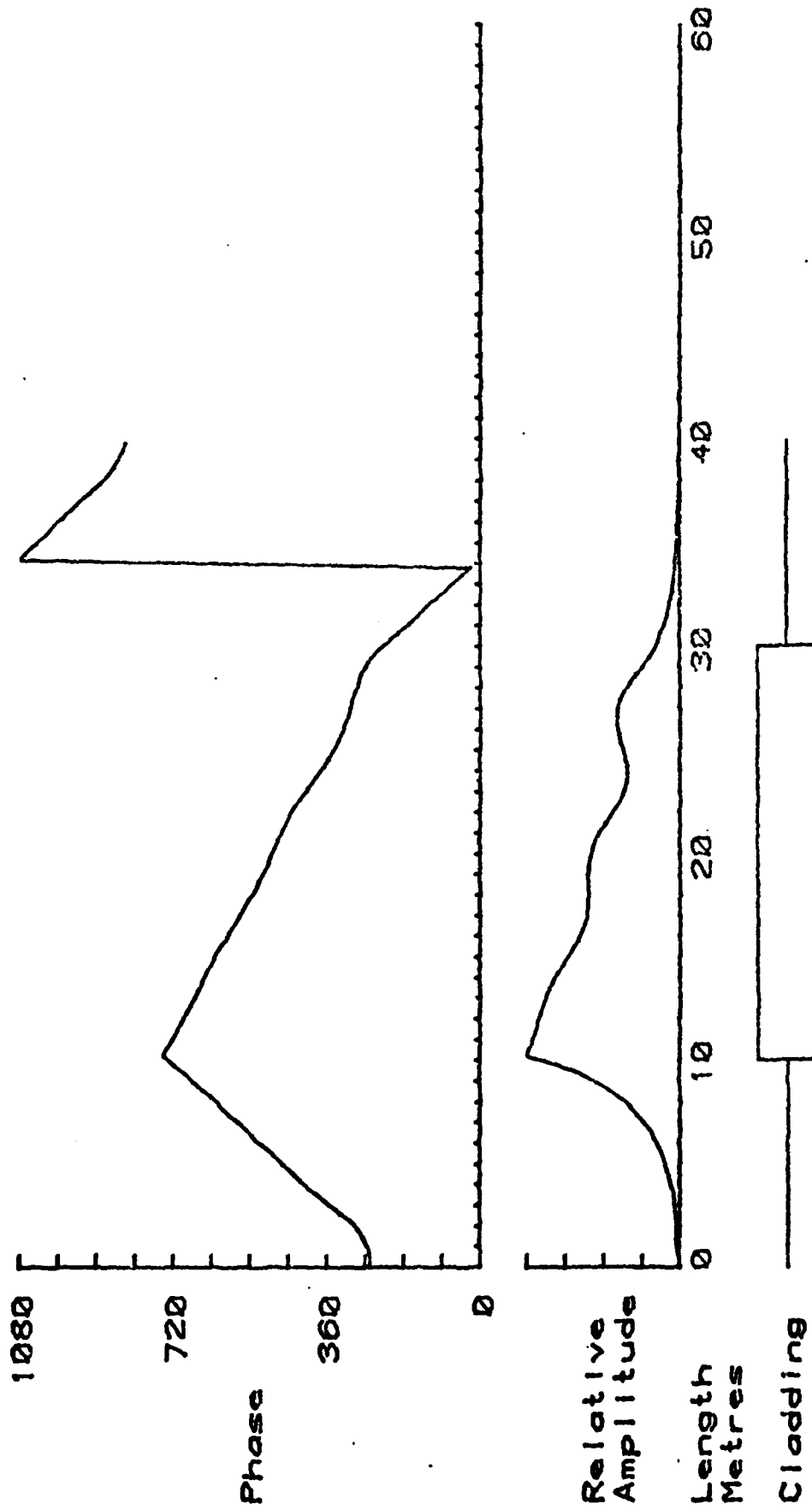


Fig. 6 11 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio= 6.6 dB. Field for 1 mw= 2.2 micro V/metre Impedance= 589.0 -242.0j

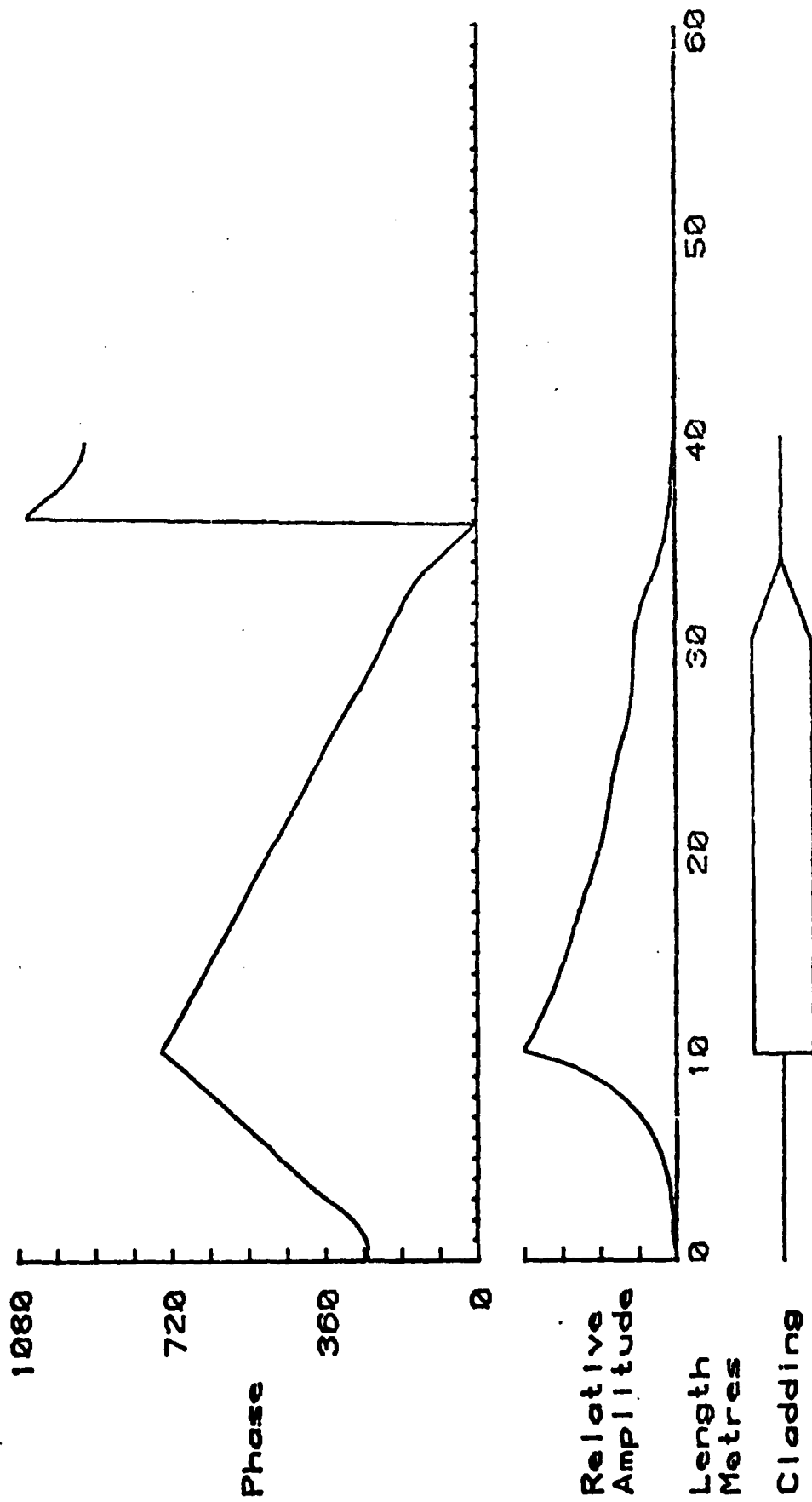


Fig. 7 11 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio= 4.9 dB. Field for 1 mw= 1.9 micro V/metre Impedance= 576.0 -246.0j

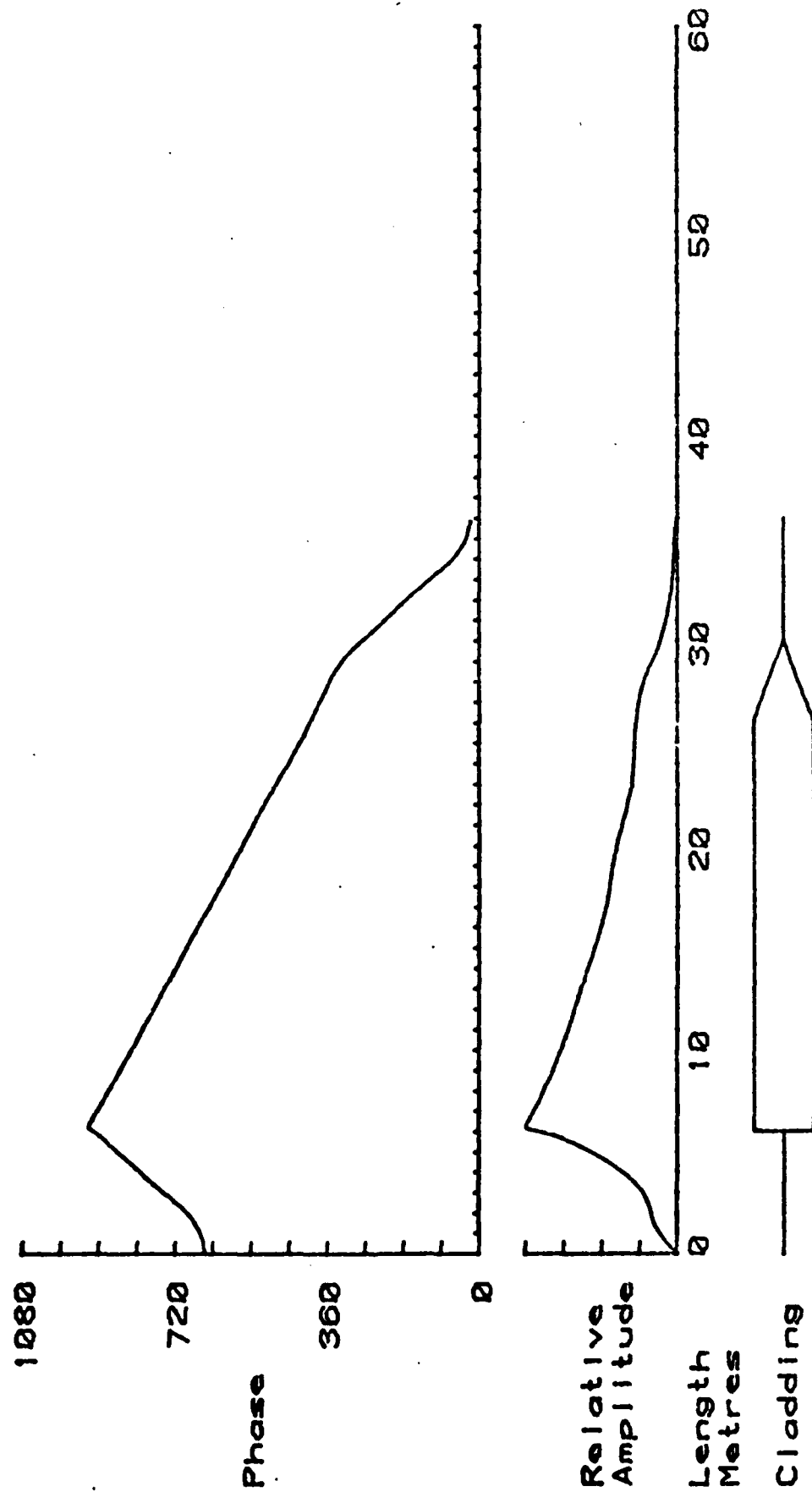


Fig. 8 11 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio= 4.3 dB. Field for 1 mw= 1.9 micro V/metre Impedance= 579.0 -244.0j

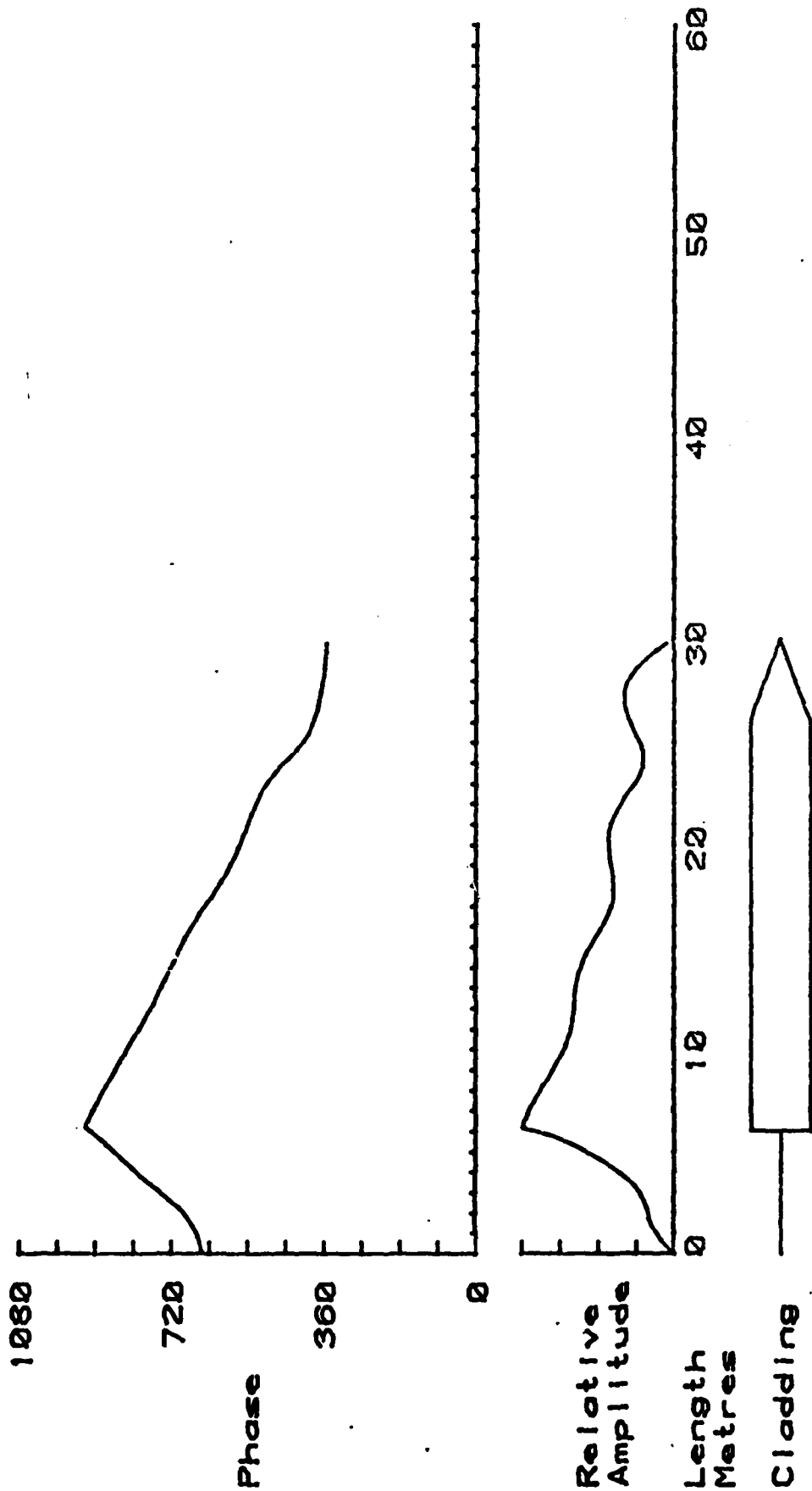


Fig. 9 11 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio= 3.5 dB. Field for 1 mW= 1.7 micro V/metre Impedance= 513.0 -237.0j



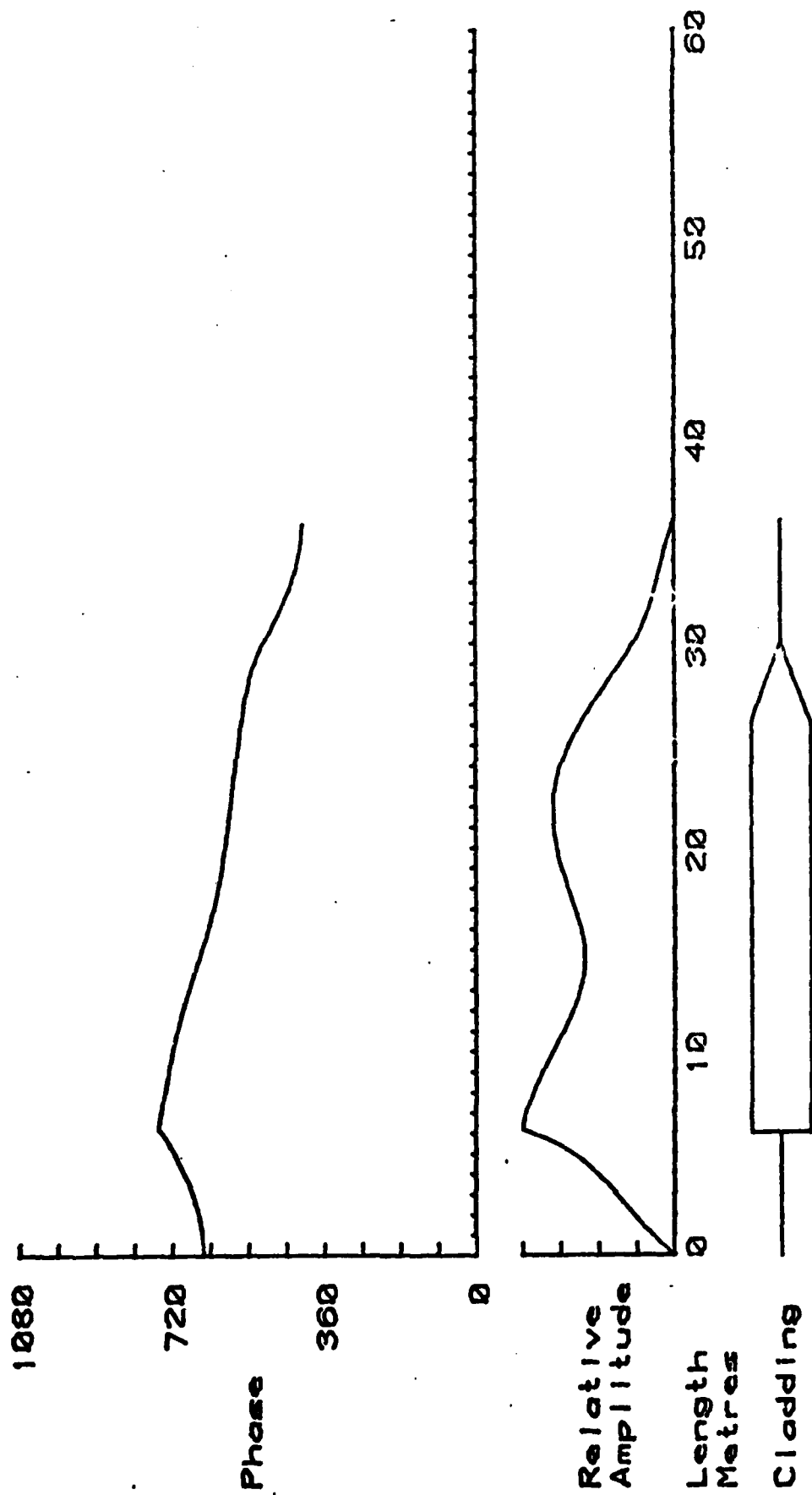


Fig. 10 4 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio=16.4 dB. Field for 1 mw= 6.9 micro V/metre Impedance= 701.0 -408.0j

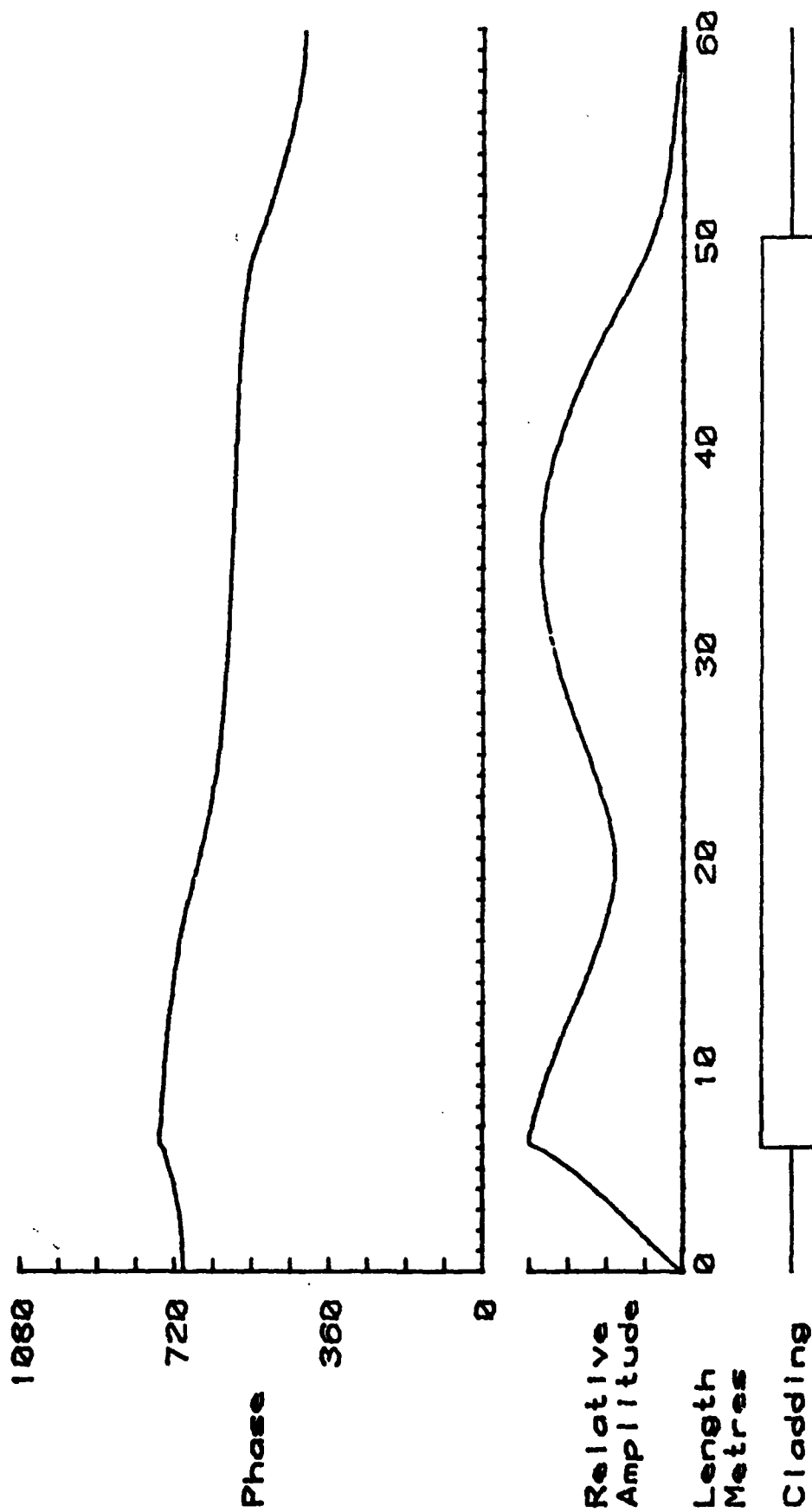


Fig. 11 2 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio=13.1 dB. Field for  $i_{mw} = 6.0$  micro V/metre Impedance= 896.0 -592.0j

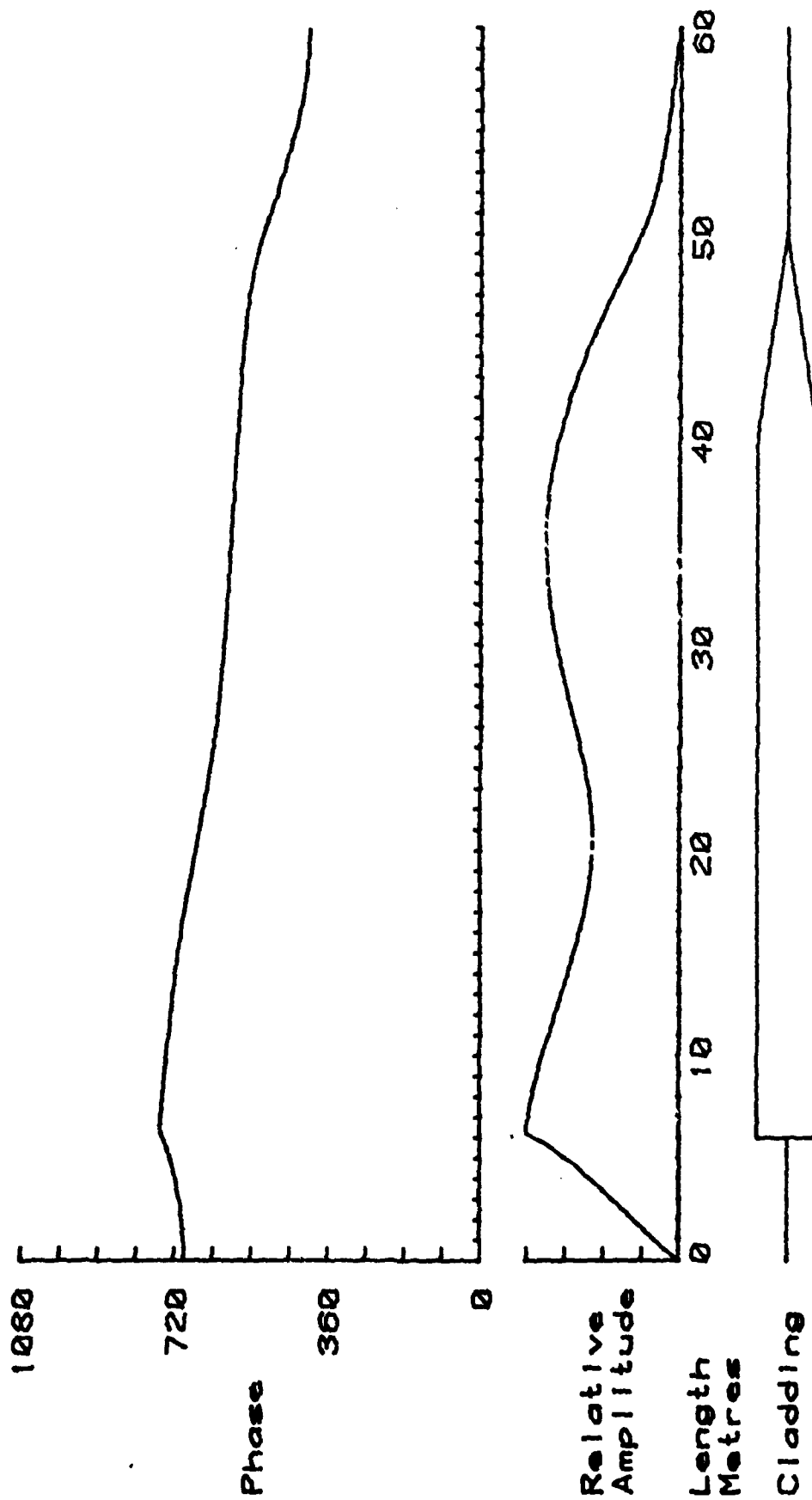


Fig. 12 2 MHz amplitude and phase calculated by the Moment Method. Cladding 5 mm tapered to zero F/B Ratio=19.7 dB. Field for 1 mw= 6.1 micro V/metre Impedance= 926.0 -578.0j

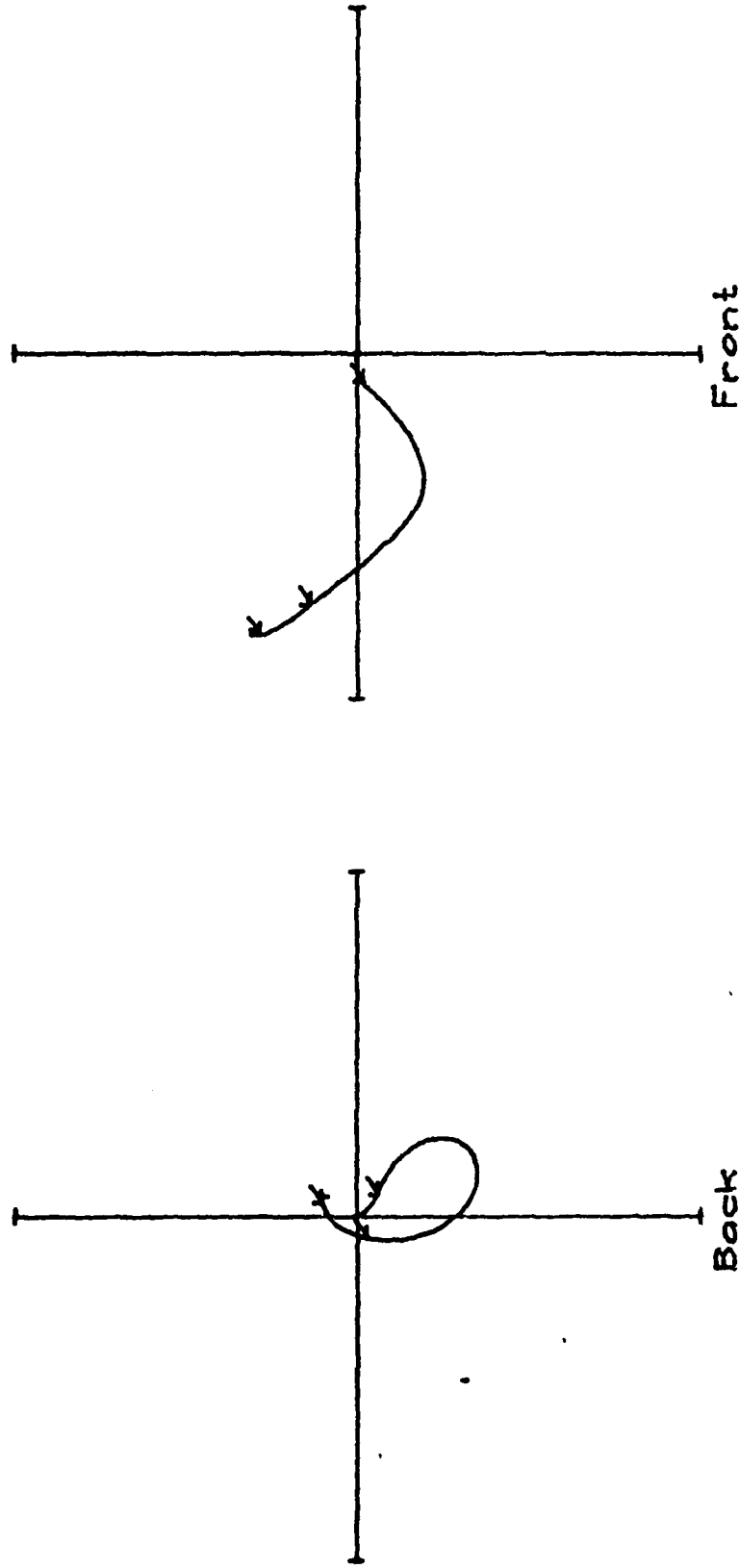


Fig. 13 2 MHz amplitude and phase diagram for for field front and backward directions as the effect of successive elements further down the line are included. Arrows show the driving point, start and finish of the taper. F/B Ratio=19.7 dB.

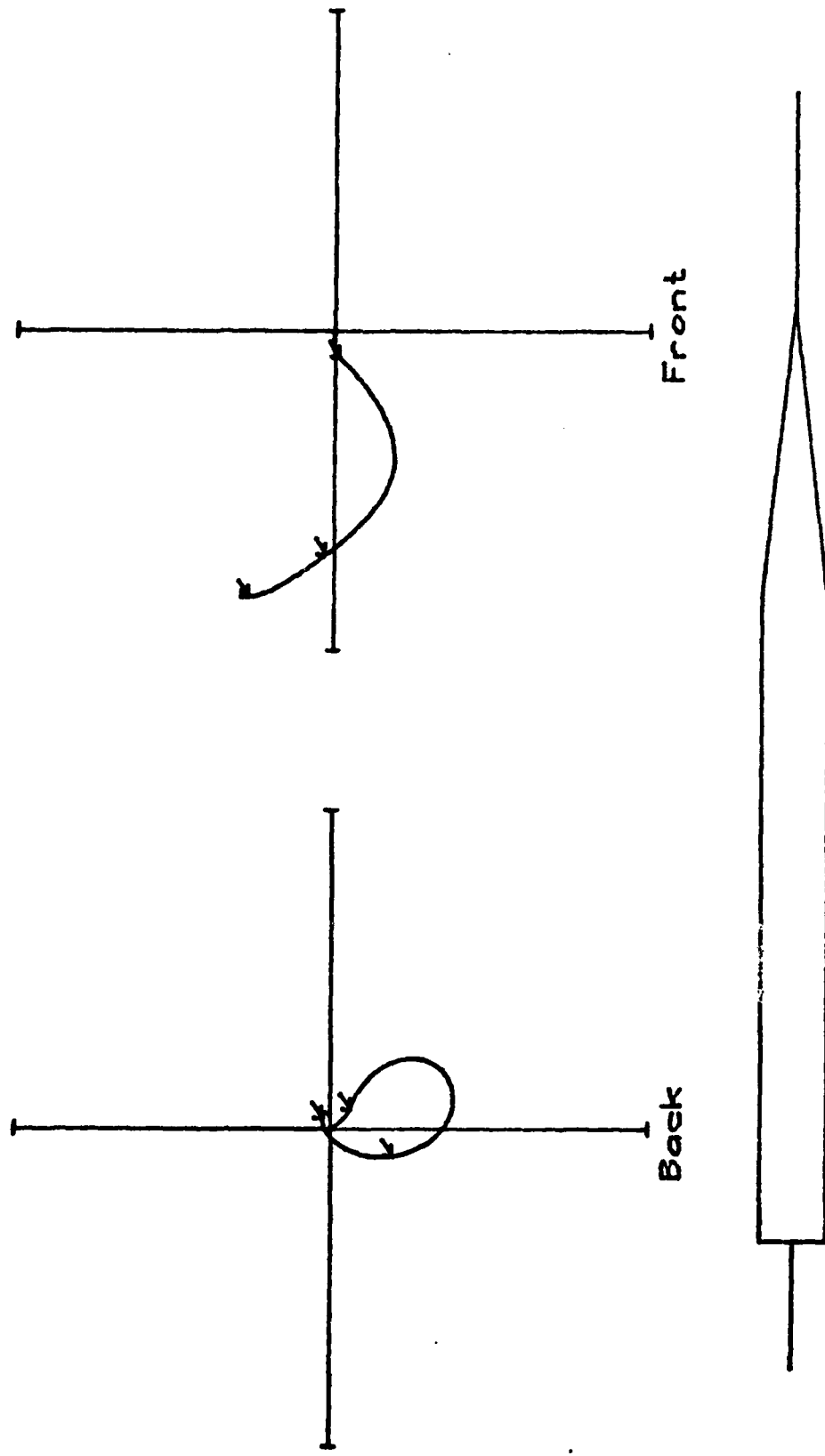


Fig. 14 2 MHz amplitude and phase diagram for for field front and backward directions as the effect of successive elements further down the line are included. Arrows show the driving point, start and finish of the taper. F/B Ratio=25.1 dB.

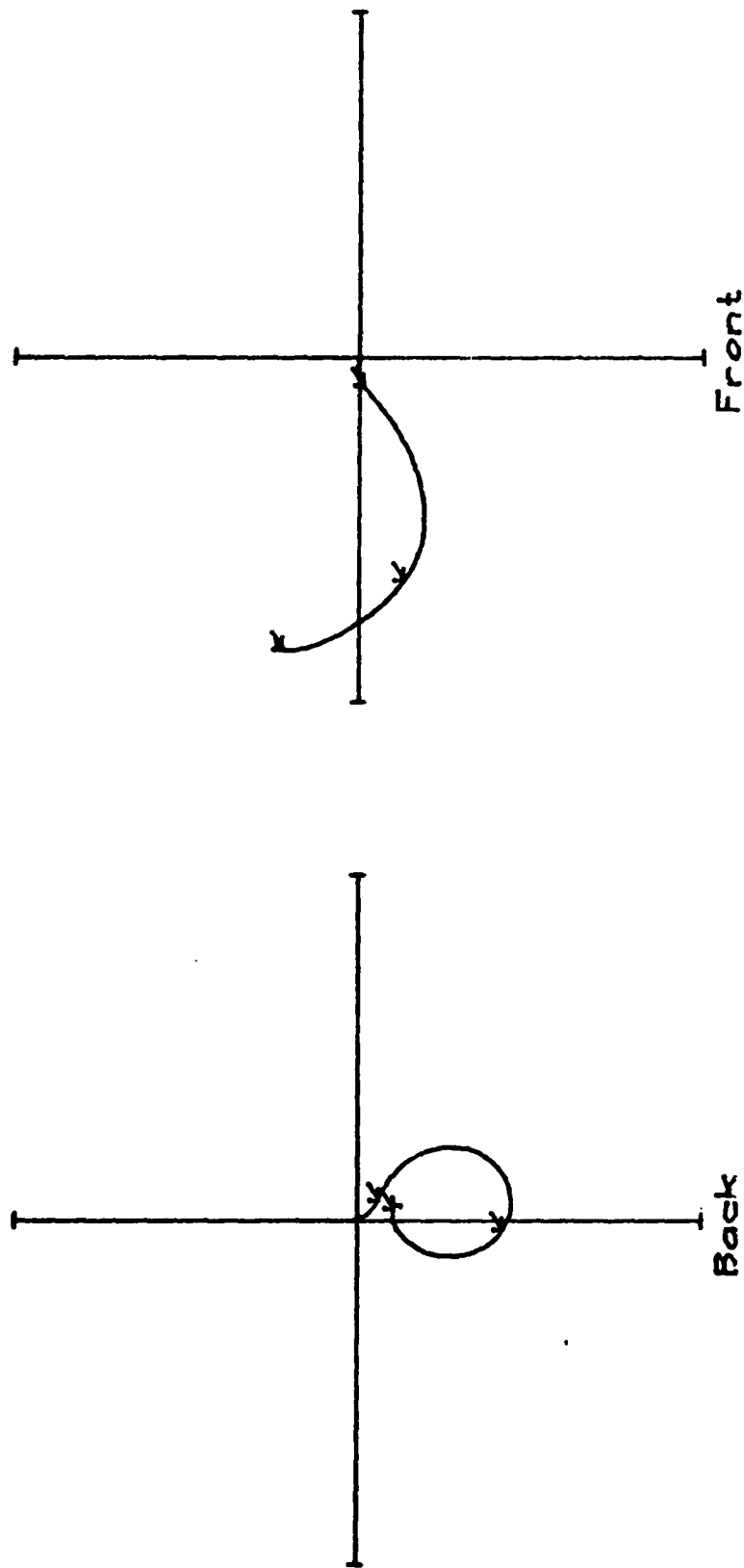


Fig. 15 2 MHz amplitude and phase diagram for for field front and backward directions as the effect of successive elements further down the line are included. Arrows show the driving point, start and finish of the taper.  
 $F/B \text{ Ratio} = 15.8 \text{ dB.}$

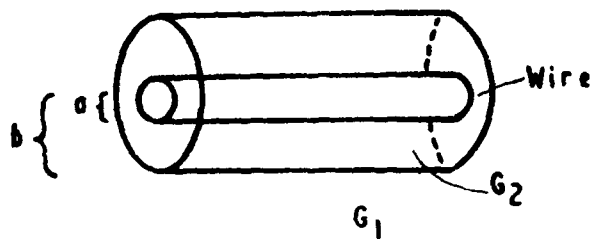


FIG. A1

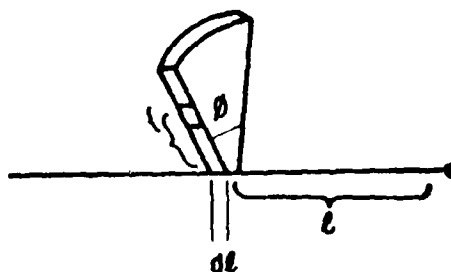


FIG. A2